Curcuma alismatifolia GAGNEP. LEAF MODELING THE PEROID OF VEGETATIVE GROWTH

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Abstract

Thegrowth of plant in a period of time can be explained by using a mathematical growth model including the exponential growth and logistic growth. This model can be used forgrowth of plant such as height, length, size or weight, depending on the researchpurpose. This model is only used for the growth of plant without considering the relations of internal activities in the plant. The research proposed this mathematical growth model forCurcuma leaf by considering on relation of the growth in leaf length of each Curcuma leaf. The result showed that during vegetative growth, there were four different stages; the first stage of growth was a bi-exponential growth model, the second stage was a bi-logistic growth model, the third stage was in reduction phase of leaf length as a bi-exponential model, and the fourth stage was in reduction phase of leaf length as a logistic growth model. The average root mean square error (RMSE) and standard error (SE) were used for accuracy assessment and both values were small errors; RMSE is about 0.06-0.21 and SE is about 0.004-0.04.

Keyword: plant growth, exponential growth model, logistic growth model, *Curcuma alismatifolia*gagnep.

Introduction

Plant growth means changes occur in weight, size, and number of plant under appropriate environmental condition that could be measured from its height, length, number, size and weight, depending on the objective of the study. The values obtained from the measurement could be built a mathematical model which could be used in explaining the behavior of plant growth or called a Growth Model. This mathematical model is basically in the form of ordinary differential equation derived from exponential growth model and multi-exponential growth model (Farina, 2002), then modified further to be the logistic growth modelby forming new algebraic format in order to explain the plant growth more effectively(Jitpattanakul,2003, Kuang, 2004, Tsoularis, 2002). Further development in growth model is Bi-Logistic growth model (Meyer, 1994, Wardhani, 2010)and Multi-Logistic growth model which can explain the growth better than the Logistic growth for simple growth and complicated growth(Meyer,1999). The study on the growth of plant leaf is important since plant leaf is an important organ in producing nutrient for plant. This research was studied on the growth of plant leaf by using Curcuma as a case study of the progress of the growth of various monocotyledon which normally has the lower leaves in small size and will wither when they are affected by surrounding condition or from the senescence of that plant. The cause of leaf senescence is from the nutrient transferring in the parts that have higher competition such as young leaf, fruit and root, etc. Therefore, this research focuses on the study for seeking mathematical model which could explain the growth of Curcuma leaf by considering the relations of behavior inside Curcuma in order to forecast the growth behavior of each leaf of Curcuma.

Methodology

Grow Curcuma in soil pot condition (17 black plastic bags) under nursery control, one plant per one bag. Measure the length of leaf (in centimeter) from each bag and record the length of each leaf everyday until withering. Take all the data to analyze and seeking the relations of each leaf growth by considering the relations of behavior within the plant. Construct the mathematical growth model using multi-exponential model or/and multi-logistic model to get the best fit on the obtained relations.

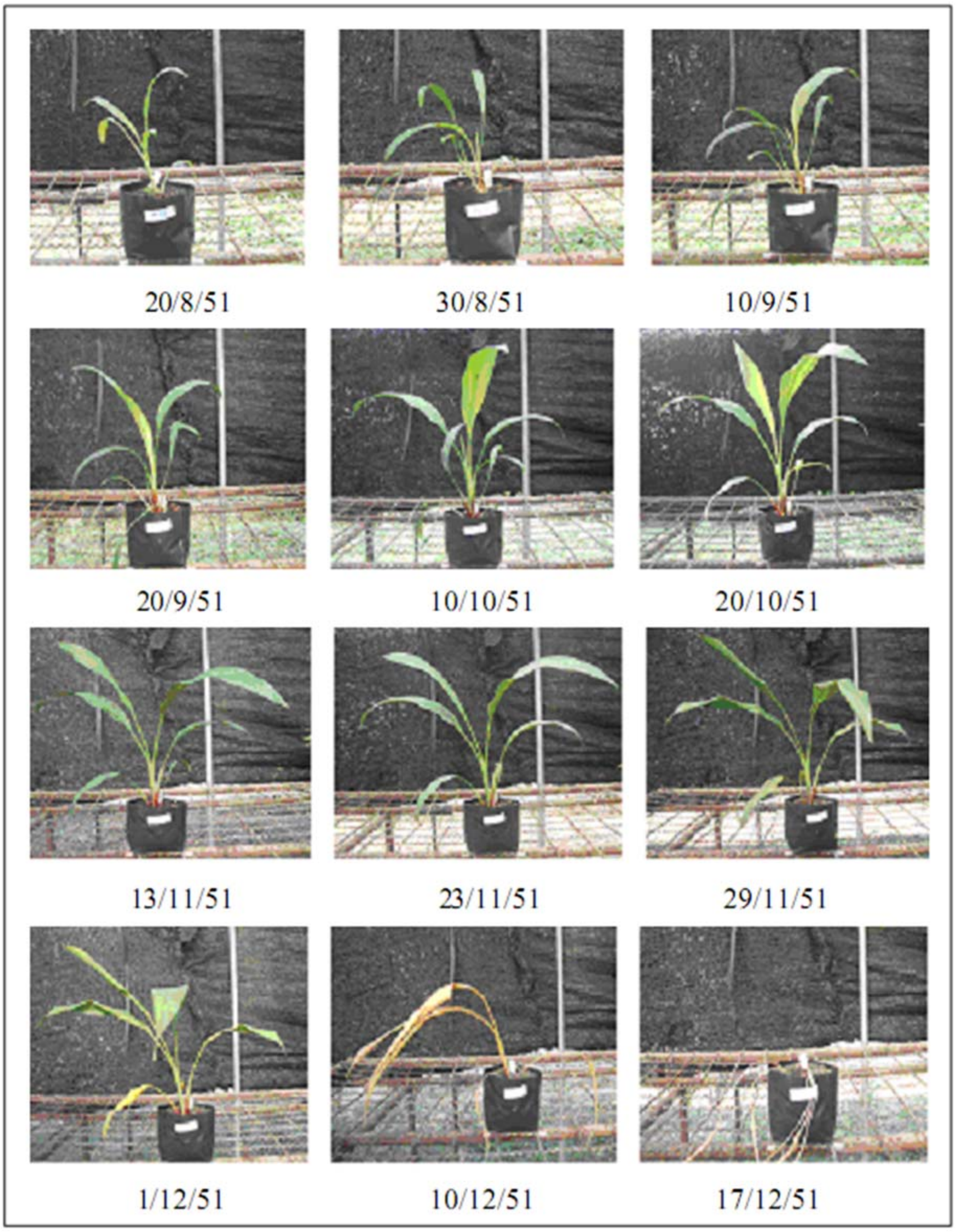

Figure 1 Curcuma leaf in the period of vegetative growth

Results

17 Curcuma were grown and recorded for 122 days as seen some pictures in Figure 1. The mathematical model was derived from the obtained data andcan be formulated in the forms of mathematic equation as presented below.

Multi Growth Model

$$
G_1(t) = \begin{cases}
\left[\beta_1 \exp\left(r_1(t-1)\right)\right] & , \ 1 \le t \le t_{1Max} \\
\left[\beta_2 \exp\left(r_2((t-t_{1Max})-1)\right)\right] & , \ t_{1Max} < t \le t_{2Max} \\
\left[\dfrac{K_1}{1+\exp\left(-\dfrac{\ln 81}{\Delta t_1}((t-t_{2Max})-t_{m1})\right)} - \left[\left(\dfrac{\ln L_{22}-\ln L_{21}}{t_{22}-t_{21}}\right)\left(t_{mL_{21}}\right)\right]\right] & , \ t_{2Max} < t \le t_{3Max} \\
\left[\dfrac{K_2}{1+\exp\left(-\dfrac{\ln 81}{\Delta t_2}((t-t_{3Max})-t_{m2})\right)} - \left[\left(\dfrac{\ln L_{24}-\ln L_{23}}{t_{24}-t_{23}}\right)\left(t_{mL_{22}}\right)\right]\right] & , \ t_{3Max} < t \le t_{4Max} \\
\left[\beta_3 \exp\left(r_3((t-t_{4Max})-1)\right)\right] & , \ t_{4Max} < t \le t_{5Max} \\
\left[\beta_4 \exp\left(r_4((t-t_{5Max})-1)\right)\right] & , \ t_{5Max} < t \le t_{6Max} \\
\left[\dfrac{K_3}{1+\exp\left(-\dfrac{\ln 81}{\Delta t_3}(t-t_{6Max})-t_{m3})\right)}\right] & , \ t_{6Max} < t \le t_{7Max}
\end{cases}
$$

$$G_2(t)=\begin{cases}\left[\beta_5\exp\left(r_5(t-1)\right)\right] & ,\ 1\le t\le t_{1Max}\\ \left[\beta_6\exp\left(r_6((t-t_{1Max})-1)\right)\right] & ,\ t_{1Max}<t\le t_{2Max}\\ \left[\dfrac{K_4}{1+\exp\left(-\dfrac{\ln 81}{\Delta t_4}((t-t_{2Max})-t_{m4})\right)}-\left[\left(\dfrac{\ln L_{32}-\ln L_{31}}{t_{32}-t_{31}}\right)\left(t_{mL_{31}}\right)\right]\right] & ,\ t_{2Max}<t\le t_{3Max}\\ \left[\dfrac{K_5}{1+\exp\left(-\dfrac{\ln 81}{\Delta t_5}((t-t_{3Max})-t_{m5})\right)}-\left[\left(\dfrac{\ln L_{34}-\ln L_{33}}{t_{34}-t_{33}}\right)\left(t_{mL_{32}}\right)\right]\right] & ,\ t_{3Max}<t\le t_{4Max}\\ \left[\beta_7\exp\left(r_7((t-t_{4Max})-1)\right)\right] & ,\ t_{4Max}<t\le t_{5Max}\\ \left[\beta_8\exp\left(r_8((t-t_{5Max})-1)\right)\right] & ,\ t_{5Max}<t\le t_{6Max}\\ \left[\dfrac{K_6}{1+\exp\left(-\dfrac{\ln 81}{\Delta t_6}(t-t_{6Max})-t_{m6}\right)}\right] & ,\ t_{6Max}<t\le t_{7Max}\end{cases}$$

$$G_3(t)=\begin{cases}\left[\beta_9\exp\left(r_9(t-1)\right)\right] & ,\ 1\le t\le t_{1Max}\\ \left[\beta_{10}\exp\left(r_{10}((t-t_{1Max})-1)\right)\right] & ,\ t_{1Max}<t\le t_{2Max}\\ \left[\dfrac{K_7}{1+\exp\left(-\dfrac{\ln 81}{\Delta t_7}((t-t_{2Max})-t_{m7})\right)}-\left[\left(\dfrac{\ln L_{42}-\ln L_{41}}{t_{42}-t_{41}}\right)\left(t_{mL_{41}}\right)\right]\right] & ,\ t_{2Max}<t\le t_{3Max}\\ \left[\dfrac{K_8}{1+\exp\left(-\dfrac{\ln 81}{\Delta t_8}((t-t_{3Max})-t_{m8})\right)}-\left[\left(\dfrac{\ln L_{44}-\ln L_{43}}{t_{44}-t_{43}}\right)\left(t_{mL_{42}}\right)\right]\right] & ,\ t_{3Max}<t\le t_{4Max}\\ \left[\beta_{11}\exp\left(r_{11}((t-t_{4Max})-1)\right)\right] & ,\ t_{4Max}<t\le t_{5Max}\\ \left[\beta_{12}\exp\left(r_{12}((t-t_{5Max})-1)\right)\right] & ,\ t_{5Max}<t\le t_{6Max}\\ \left[\dfrac{K_9}{1+\exp\left(-\dfrac{\ln 81}{\Delta t_9}(t-t_{6Max})-t_{m9}\right)}\right] & ,\ t_{6Max}<t\le t_{7Max}\end{cases}$$

$$G_4(t) = \begin{cases} \left[\beta_{13}\exp\left(r_{13}(t-1)\right)\right] & , \ 1 \le t \le t_{1Max} \\ \left[\beta_{14}\exp\left(r_{14}((t-t_{1Max})-1)\right)\right] & , \ t_{1Max} < t \le t_{2Max} \\ \left[\dfrac{K_{10}}{1+\exp\left(-\dfrac{\ln 81}{\Delta t_{10}}((t-t_{2Max})-t_{m10})\right)} - \left[\left(\dfrac{\ln L_{52}-\ln L_{51}}{t_{52}-t_{51}}\right)\left(t_{mL_{51}}\right)\right]\right] & , \ t_{2Max} < t \le t_{3Max} \\ \left[\dfrac{K_{11}}{1+\exp\left(-\dfrac{\ln 81}{\Delta t_{11}}((t-t_{3Max})-t_{m11})\right)} - \left[\left(\dfrac{\ln L_{54}-\ln L_{53}}{t_{54}-t_{53}}\right)\left(t_{mL_{52}}\right)\right]\right] & , \ t_{3Max} < t \le t_{4Max} \\ \left[\beta_{15}\exp\left(r_{15}((t-t_{4Max})-1)\right)\right] & , \ t_{4Max} < t \le t_{5Max} \\ \left[\beta_{16}\exp\left(r_{16}((t-t_{5Max})-1)\right)\right] & , \ t_{5Max} < t \le t_{6Max} \\ \left[\dfrac{K_{12}}{1+\exp\left(-\dfrac{\ln 81}{\Delta t_{12}}(t-t_{6Max})-t_{m12})\right)}\right] & , \ t_{6Max} < t \le t_{7Max} \end{cases}$$

$$G_5(t) = \begin{cases} \left[\beta_{17}\exp\left(r_{13}(t-1)\right)\right] & , \ 1 \le t \le t_{1Max} \\ \left[\beta_{18}\exp\left(r_{18}((t-t_{1Max})-1)\right)\right] & , \ t_{1Max} < t \le t_{2Max} \\ \left[\dfrac{K_{13}}{1+\exp\left(-\dfrac{\ln 81}{\Delta t_{13}}((t-t_{2Max})-t_{m13})\right)}\right] & , \ t_{2Max} < t \le t_{3Max} \\ \left[\dfrac{K_{14}}{1+\exp\left(-\dfrac{\ln 81}{\Delta t_{14}}((t-t_{3Max})-t_{m14})\right)}\right] & , \ t_{3Max} < t \le t_{4Max} \\ \left[\beta_{19}\exp\left(r_{19}((t-t_{4Max})-1)\right)\right] & , \ t_{4Max} < t \le t_{5Max} \\ \left[\beta_{20}\exp\left(r_{20}((t-t_{5Max})-1)\right)\right] & , \ t_{5Max} < t \le t_{6Max} \\ \left[\dfrac{K_{15}}{1+\exp\left(-\dfrac{\ln 81}{\Delta t_{15}}(t-t_{6Max})-t_{m15})\right)}\right] & , \ t_{6Max} < t \le t_{7Max} \end{cases}$$

The model of Curcuma leaf growth can be categorized in 5 intervals and in the forms of multi-exponential growth and multi-logistic growth. $G(t)$ is the growth at a point of time, t is the time of the growth, K is the limited length size of the leaf, Δt is the period of leaf growth, t_m is the time of full leaf growth, t_{Max} is the end time of growth, t_{mL} is the time at the high growth rate, r is the rate of leaf growth, β is the initial leaf length for exponential growth behavior and L is the initial or final leaf length for logistic growth behavior.

Discussion and Conclusion

When comparing the obtained mathematical growth model with the value obtained from the experiment, the result of the model and the real data was presented in Figure 2.

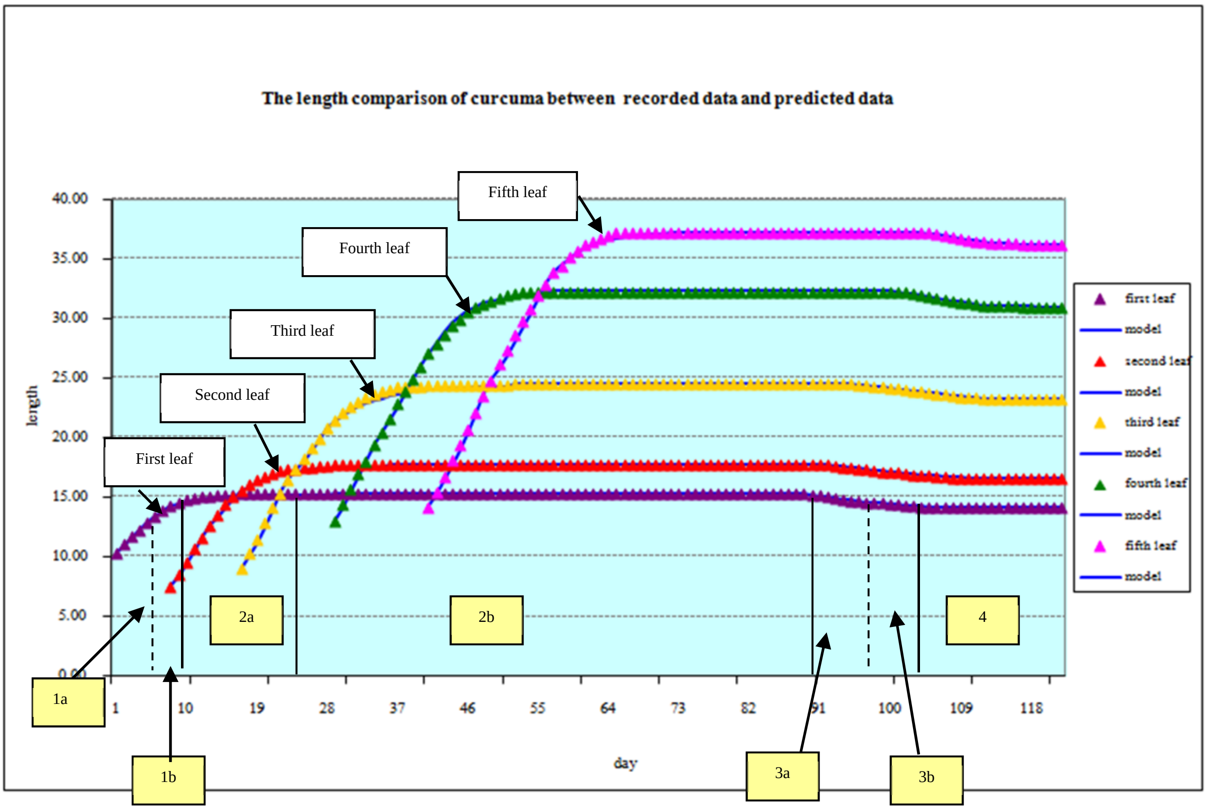

Figure 2 The Curcuma leaf length comparison between the experiment and the model

The results showed that each leaf hadthe similar characteristic curve.The period of each leaf growth of Curcuma can also be divided into 4 different stages; the first stage of growth was a bi-exponential growth model (*i.e.*,1a and 1b)); the second stage was a bi-logistic growth model (*i.e.*,2a and 2b); the third stage was reduction of leaf length as a bi-exponential model (*i.e.*,3a and 3b)and the fourth stage was reduction of leaf length as a logistic model (*i.e.*, 4). From Figure 2, the graph of the first leaf showed that at first stage (1a and 1b), the growth was rapid and unlimited giving the model in the first stage as an exponential form. After that the growth started to be constant (2a and 2b) and reached the limit satisfied with a logistic growth model. During period 3a to 3b the leaf began to wither with lower the growth rate. In this period, the leaf decayed rapidly and unlimited, like an exponential model. In the fourthstage, the withering of the leaf became constant, fitfor a logistic model. In addition, from the graph, it showedthat the first leaf slowly stopped growing when the second leaf had emerged, and the second leaf would stop growing when the third leaf emerged and the same behavior for the fourth and the fifth leaf. This revealed that the growth of each Curcuma leaf was related to one another due to food nutrient transferring. The higher competitive, young leaf needed more nutrition to grow and this would affect the slow growth of the preceding leaf. The average root mean square error (RMSE) and standard error (SE) were used for accuracy evaluation. The Root Mean Square Error (RMSE) obtained from the real data and the values from the model of each leaf were shown in Table 1.

Table 1 The average root mean square error (RMSE) and standard error (SE)

	Root Mean SquareError (RMSE)	Standard Error (SE)
1^{st} leaf	0.0632	0.0041
2^{nd} leaf	0.1134	0.0131
3^{rd} leaf	0.2009	0.0411
4^{th} leaf	0.1224	0.0153
5^{th} leaf	0.2086	0.0446

According to the small errors of RMSE and SE, the mathematical modelcan be used for explanation and prediction in the growth of Curcuma leaf.

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